

A Simple Computer Game for Learning Mathematics

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Abstract

It is sometimes unquantifiable how hard it is for most people to deal with game addiction. Several articles have equally been published to address this subject, some suggesting the concept of Educational and serious games. Similarly, researchers have revealed that it does not come easy learning a subject like math. This is where the illusive world of computer games comes in. It is amazing how much people learn from games. In this paper, we have designed and programmed a simple PC math game that teaches rudimentary topics in mathematics.

Keywords: e-learning; Educational and Serious Games, Learning Math

1. Introduction

Games can be incredible, especially when learning so much in the face of anxiety and pleasure. Games are more than mere software (Funge, 2004). They are often the combination of music, art and creative imaginations (Deleon, 2010). Bethke (2003) argued two reasons why one should make a game: to share a dream and to teach. Every form of game teaches one thing or the other. Some games even teach mathematics explicitly (GofM, 2010). Over time, developers have created a number of math-teaching games such as Mixed-Up Math, NumSkull Math (Kprobe, 2010), and Game of Math3 (GofM, 2010).

Researchers have also studied the effects of educational games on Math Achievements (Aronowitz, 2009). Further research on why people play video games was carried out by Amory et al (1998) and Malone (1981). Malone identified three main reasons why people play games: fantasy, challenge and curiosity. Further research confirms these findings; for example, in experiments using educational software, Amory et al (1998) identified curiosity as a common motive in playing a game. Some other articles have been published suggesting that the relationships between video games and digital learning technologies is worthy of more investigation (Kirriemuir, 2002).

Kirriemuir (2002) stated that games are increasingly used to support teaching and learning e.g., using text adventures to assist in teaching English as second language. Conclusions as to the effectiveness of games for educational purposes differ; one particular review of relevant research indicated that mathematics was a subject where the use of games was usually superior to traditional classroom instructions (Kirriemuir, 2002).

The challenge in this work is to present mathematics in the form of a PC Game for ease of learning and acquaintance to the subject. Our targeted audience is every mathematics students (age and academic level notwithstanding). However, finding solutions to the phobia for mathematics amongst its students have been the focus of past studies in educational management around the world (Johnston-Wilder et al, 2002). Looking inwardly, this problem is also prevalent in the Nigerian Educational System (NMC-MIP, 2006). Several suggestions and solutions have equally been provided. However, most of the offered solutions to this problem are neither sufficient nor effective and at worse, less implemented in most scenarios (NMC-MIP, 2006).

With that said, as percepts on our community (Nigeria), we observed the game addiction of our students and therefore based our improvement strategy on it. The PC game developed in this work was named math-magic. We have attempted to delude our players with the world of magic. They will perceive themselves as magicians by casting math spells and in turn, they will earn some money (virtual money) for successes recorded during the game play. This PC game was implemented with the .net framework 4.0 and hence will suffice extensively since students should find mathematics more approachable and interesting if it is presented as a computer game.

The remaining of this article is organized as follows: section 2 reviews educational games, section 3 presents the game design, section 4 attempts to provide a model for valid math problems generation to be supplied to the game as puzzles at real time, section 5 has the screenshots from the game and section 6 gives a concise conclusion.

2. Educational Games

Educational games are games that have been specifically designed to teach people about a certain subject, expand concepts, reinforce development, understand an historical event or culture, or assist them in learning a skill as they play. They include board, card, and video games (Dostal, 2009).

Educational video games are known for offering edutainment because they combine education and entertainment (Dostal, 2009). Closely related to the use of educational games is the use of what is known as serious games. An educational computer game can be defined as an electronic medium with all the characteristics of a gaming environment that have intended educational outcomes targeted at specific groups of learners (Aronowitz, 2009).

Video games can aid the development of proficiency by allowing users to interact with objects and manipulate variables. They are said to be particularly effective when designed to address a specific problem or teach a certain skill in curriculum subjects, where specific objectives can be stated and when deployed selectively within a context relevant to the learning activity and goal (Dickheiser, 2003).

Simple types of games can be designed to address specific learning outcomes such as recall of factual content. For instance, the Nobel Prize Foundation website uses on-line games to aid children in understanding the discoveries made by its laureates by embedding the scientific knowledge as part of the game environment (nobelprize, 2010).

To aid in educating students and adults about the finer details of different political systems, numerous companies have developed simulations that immerse the player into different political systems by forcing them to make realistic political decisions. These games vary from running an actual election campaign to games that allow the player to make the day-to-day decisions of running a country, as seen in Democracy (a game). These types of games are targeted at students, educators and adults alike (Kirriemuir, 2002).

2.1 Mathematical Game or Puzzle?

Literature has exposed specific controversies in the use of these terms in recent times. A mathematical game is a game whose rules, strategies, and outcomes can be studied and explained by mathematics. Examples of such games are Tic-tac-toe, Dots and Boxes. On the surface, a game need not seem mathematical or complicated to be a mathematical game. For an example, even though the rules of Mancala are straightforward, mathematicians analyze the game using combinatorial game theory.

Mathematical games differ from mathematical puzzles in that all mathematical puzzles require math to solve them whereas mathematical games may not require the knowledge of mathematics to play them or even to win them. Thus the actual mathematics of mathematical games may not be apparent to the average player.

Some mathematical games are topics of interest in recreational mathematics. Deductively, the term “mathematical games” is used to refer to games with mathematical designs. In our case, even though the proposed game is a mathematical game, it is important to state here that, serious mathematical puzzles/problems will be solved by the gamers at real-time. Hence, we shall be using these terms interchangeably in this work.

When studying the mathematics of games, the mathematical analysis of the game is of paramount importance than actually playing the game (Champanard, 2003). To analyze a game mathematically, the mathematician studies the rules of the game in order to understand the inner-workings of the game, to determine winning strategies, complexities and to possibly determine if a game has a solution.

3. Designing Math-magic

Game design deals with content and rules definition. The name math-magic, given to our new game is from the concatenation of the prefix “math” (from mathematics) and magic. In this paper, we refer to our players as math magicians hereby implying that they have the magic to get math problems solved.

3.1 Story Development, Game Play and Scoring System

The storyline of our game is a straightforward one. Our player is made to believe that he/she is a sorcerer sent from the future to avert certain evil using magic. In this case, the magic is math. The player will have a limited time to get the math problems solved and could apply a variety of moves to get the problem solved. The following are the moves a player could use.

- Move One: Solve the math problem and avert evil before the timer runs out and earn some money for doing so.
- Move Two: Hire a smarter sorcerer (within the game) to cast the spell and avert the evil on your behalf. This move will require the player to pay some amount of money and as such it could be used by players that must have made some money from successive plays.

The game will also provide the players with a gallery of sorcerers with varying degree of expertise from which a player may employ and pay any expert of their choice to get the problems solved for them. The design uses probability to evaluate the prowess of these sorcerers.

The scoring system addresses the questions: who gets what and how. As described in the analogy of Sun (2008), Scoring Systems are found in almost every game out there. They are a key component in games. The idea of scoring is relatively simple. Whenever the player accomplishes a task within the game, they are rewarded with points to add to their score. However, points are not limited to numeric scores; they could also be experience points, money or other valuables. As the player accomplishes tougher tasks, he/she gains more points. This is generally how scoring systems work in most games. The unit of our scoring system is in money (dollars). Designing with money as points is intended to enhance the gameplay because everybody wants to earn money even in the real-world sense.

4. Modeling Math Problems for the game

The mathematical problems fashioned for the game covers elementary topics in mathematics. The table below shows the difficulty levels of the game and the associated math topics respectively.

Level	Stage	Problem Family
1 Math-Novice	1	Simple Arithmetic (+, -, /, X), Simple & Compound Interest Logic, and Linear Evaluations, Brain Teasers, Word Problems
	2	Simple Evaluations, Indices and Logarithms, Primes, Time calculations
	3	Quadratic Equations (Real Roots), Trigonometric Computations, Probability
2 Math-Professional	4	Progressions (AP, GP), Recursive Sequences (Fibonacci, Ackermann e.t.c)
	5	Variation, Surds and Quadratic Equations (Complex Roots)
	6	Simultaneous Equations
3 Math-Pundit	7	Factorials, Combinatorial Analysis, Binomial Expansion
	8	Matrices Arithmetic and Determinants
4 Math-Legend	9	Numerical Iterations, Complex Quadratic Equations

Table 1. Difficulty Levels and Math-Topics Covered

Incorporating complexity measure when testing students on math problems is of paramount importance (Johnston-Wilder et al, 2002), therefore our implementation has ensured that by a number of methods such as varying the associated constants. For instance, when solving quadratic problems, the complexity definitely increases as the constant becomes more scrambled. Let us compare the two succeeding equations:

(a). $x^2 + 4x + 4 = 0$

(b). $\frac{4}{19}x^2 + 0.54x + \frac{108}{97} = -\frac{3}{7}$

the equation in (a) has an intuitive answer because of the values of a, b and c (from $ax^2 + bx + c = 0$) and by mere inspection, a problem solver of 'average' intelligence will rarely require further calculations to deduce that the roots of the equation are (2, 2). On the contrary, in (b), even the smartest person will require some evaluation of certain degree to arrive at the roots. Conclusively, we have used this constant manipulation technique to maintain a range of difficulty design, scrambling the constants as the level increases.

We have also isolated the modules that handle the supply of these problems to the game to facilitate easy update or patch rendering. Consequently, other area of mathematics can be incorporated in the game in subsequent versions or updates.

On the surface, lots of our anticipated gamers (likely to be students) might perceive finding solutions to elementary mathematical problems as trivial. This impression is totally relative to individuals with varying mathematical backgrounds/strength and at worse, could be completely wrong when one encounters non-trivial problems that require hard thinking to crack or when there is a time constrain to solve a problem.

Hence, our design of the math problems are such that the ability of a player is trapped somewhere along the line between the first stage and the last stage of the game play by increasing the complexity of the problems and simultaneously reducing the time allowed. In situations where this is impossible, then we say the player has successfully completed every difficulty level but still hope to trap him with a future update or version of the game, consisting of tougher math problems.

In this subsection, we will describe our problem formulation techniques with specific examples and written algorithms.

Example 1: Model for generating quadratic problems

Still on our quadratic example, supposing we are to formulate a model for generating quadratic problems for our game at runtime and considering the general form for quadratic equations:

$$ax^2 + bx + c = 0 \quad (\text{E4.1})$$

taking α and β as the roots of the equation in (E4.1), it suffices to say that:

$$x^2 - (\text{sum})x + \text{product} = 0 \quad (\text{E4.2})$$

$$\text{Where } \text{sum} = \alpha + \beta = -\frac{b}{a} \quad \text{and } \text{product} = \alpha \cdot \beta = \frac{c}{a}$$

Hence, we deduce that

$$x^2 - (\alpha + \beta)x + \alpha \cdot \beta = 0 \quad (\text{E4.3})$$

Now, all we need is random values for the variables: α and β , therefore, assuming we choose to define a domain for α and β respectively (for a trivial case)

$$\alpha \in [3, 6] \text{ and } \beta \in [-2, 5]$$

closed intervals, and then call the $\text{rand}()$ function to get random values for α and β in the range. Assuming that the rand function gave us $\alpha = 4$ and $\beta = -1$, then from (E4.3) we have,

$$x^2 - 3x - 4 = 0$$

Which has the solution,

$$(x - 1)(x + 4) = 0 \Rightarrow x = 1 \text{ or } -4$$

We then attempt to store these constants (4 and -1) in the memory of the host machine for the reference of the game so as to avoid recurrence.

The above example is quite intuitive and hence we present an algorithm (written in SPARKS) that expresses it accordingly:

```

Procedure getQuadProblem(upper_bnd, lower_bnd, playerID)
  Parameters integer upper_bnd, lower_bnd, playerID
  Real alpha, beta:
  do
    alpha ← Rand(upper_bnd, lower_bnd)
    beta ← Rand(upper_bnd, lower_bnd)
    // program checks database if player has seen this
    // problem before and loops if otherwise
    loop until (problem_is_familiar(alpha,beta)==false)
    // continues to problem formulation
    sum ← alpha + beta
    prod ← alpha * beta
    equation ← Concat('x^2','+', (-1) * sum, 'x +', prod, '=0')
    //program should learns that player has seen this problem
    call store_to_db(playerID, alpha, beta)
    return equation
End Procedure

```

The above algorithm was coded into a VB.NET method that generates (as many as needed) random quadratic problems for the gamer to solve at run time. The table below shows few problems generated within the game.

$7(k^2) + 7k - 504 = 0$	$- 1(t^2) - 1t + 30 = 0$	$- 8(p^2) + 24p + 32 = 0$	$- 2(m^2) + 14m - 24 = 0$
$- 6(h^2) + 12h + 210 = 0$	$- 6(m^2) + 36m + 42 = 0$	$7(y^2) + 84y + 252 = 0$	$5(h^2) - 20h - 160 = 0$
$- 4(c^2) + 8c + 320 = 0$	$- 7(q^2) - 63q - 126 = 0$	$- 4(k^2) - 44k - 96 = 0$	$3(t^2) - 9t - 162 = 0$
$- 5(q^2) + 70q - 225 = 0$	$4(y^2) - 28y + 40 = 0$	$7(b^2) + 14b - 56 = 0$	$- 1(f^2) - 4f + 45 = 0$
$- 4(x^2) + 20x - 24 = 0$	$- 4(e^2) - 4e + 24 = 0$	$5(r^2) - 75r + 270 = 0$	$7(q^2) + 42q + 56 = 0$
$- 2(e^2) - 18e - 40 = 0$	$3(p^2) + 9p - 120 = 0$	$7(f^2) - 77f + 70 = 0$	$7(b^2) - 77b + 196 = 0$
$- 6(p^2) + 24p + 270 = 0$	$- 6(x^2) - 30x + 84 = 0$	$- 6(c^2) + 12c + 480 = 0$	$- 7(c^2) - 91c - 210 = 0$
$- 2(w^2) - 10w + 12 = 0$	$7(n^2) + 63n - 70 = 0$	$7(f^2) - 112f + 420 = 0$	$7(e^2) - 14e - 245 = 0$
$7(q^2) - 49q + 70 = 0$	$8(w^2) + 104w + 336 = 0$	$- 9(t^2) + 18t + 315 = 0$	$5(m^2) + 20m - 25 = 0$
$- 2(p^2) - 2p + 24 = 0$	$- 8(p^2) + 8p + 336 = 0$	$4(k^2) - 4k - 360 = 0$	$8(w^2) + 16w - 64 = 0$
$7(a^2) + 112a + 441 = 0$	$- 2(w^2) + 2w + 12 = 0$	$5(n^2) + 85n + 360 = 0$	$3(k^2) + 3k - 60 = 0$
$- 7(p^2) - 42p - 35 = 0$	$- 4(t^2) + 16t + 48 = 0$	$1(r^2) - 4r - 12 = 0$	$- 2(y^2) - 16y - 30 = 0$
$5(w^2) - 20w - 300 = 0$	$- 1(r^2) + 14r - 40 = 0$	$- 2(k^2) + 8k - 6 = 0$	$- 9(w^2) - 36w + 45 = 0$
$- 6(t^2) - 24t + 270 = 0$	$- 2(n^2) - 26n - 80 = 0$	$- 5(m^2) + 25m + 30 = 0$	$- 9(b^2) + 54b - 81 = 0$
$- 9(y^2) - 153y - 648 = 0$	$3(b^2) - 39b + 108 = 0$	$8(a^2) + 88a + 80 = 0$	$8(k^2) - 48k + 64 = 0$
$7(p^2) - 35p - 168 = 0$	$- 9(k^2) + 117k - 378 = 0$	$2(p^2) - 4p - 160 = 0$	$8(q^2) + 8q - 16 = 0$
$- 6(b^2) - 6b + 36 = 0$	$- 6(r^2) + 72r - 210 = 0$	$4(p^2) + 56p + 160 = 0$	$- 2(q^2) + 2q + 60 = 0$
$- 7(b^2) + 98b - 336 = 0$	$- 3(e^2) + 54e - 240 = 0$	$- 2(h^2) - 6h + 8 = 0$	$- 7(n^2) - 21n + 280 = 0$
$1(d^2) + 4d - 60 = 0$	$5(x^2) + 25x + 20 = 0$	$- 9(a^2) + 99a - 270 = 0$	$- 9(e^2) + 36e + 189 = 0$
$- 8(e^2) - 64e + 160 = 0$	$- 5(t^2) - 10t + 40 = 0$	$2(d^2) - 30d + 108 = 0$	$- 6(h^2) + 24h - 18 = 0$
$7(w^2) - 98w + 280 = 0$	$- 4(t^2) - 48t - 144 = 0$	$7(r^2) - 49r + 70 = 0$	$8(b^2) - 8b - 720 = 0$
$- 3(p^2) + 42p - 144 = 0$	$6(w^2) - 54w + 84 = 0$	$2(w^2) + 8w - 90 = 0$	$- 4(w^2) + 48w - 80 = 0$

$8(x^2) - 16x - 384 = 0$	$- 8(y^2) + 144y - 648 = 0$	$- 8(r^2) - 72r - 144 = 0$	$7(p^2) + 21p - 196 = 0$
$6(x^2) - 42x - 180 = 0$	$8(w^2) - 32w - 96 = 0$	$- 3(d^2) + 21d - 30 = 0$	$- 2(t^2) + 8t + 90 = 0$
$- 1(k^2) + 5k + 36 = 0$	$- 7(c^2) - 21c + 196 = 0$	$- 2(e^2) - 28e - 90 = 0$	$- 1(f^2) - 18f - 80 = 0$

Table 2. A hundred randomly generated Quadratic Problems

It is important that we mention here that the problems supplied to the gamer at this stage of the game are quadratic and has real roots (complex cases were avoided during formulation) and as such, we have avoided controversial solutions since our players would have to plug in discrete answers to supplied problems.

5. Screen Shots from the Game

At game start, we inspire our players with the following inscriptions:

It is now or never!

Are you the ONE?

Is it time for the prophecy to come true?

We've been expecting the ONE who will save the world from impending doom.

Magic is the only way,

Math-Magic, we just hope you have enough of it.

The inscription stays for 5 seconds on the screen and fades off. Then the window for the first magic appears. The family of math problems to be displayed depends on the difficulty level settings within the game at that instance, but for the purpose of this demo, we stick to our quadratic example.

In this case, the player's difficulty level is Stage One, Level Three. At this level, the player will only have 120 seconds to solve every math problem that follows after he/she strikes the Enter Key. The scoring is in terms of US Dollars. The player gets paid \$50 for every right answer.

As part of motivation for the game play, the player also gets a status (a title) that is a function of the fortune made from successive game plays. For the initial start, this status will appear as "Broke Math-Magician" and subsequently, as the player makes more money from solving math problems, the status improves to reflect tags such as "Little Purse", "Money Maker", "Rich Freak", "Local Millionaire" and so on. In figure 2 above, the timer has counted down to 112 from 120, the problem to be solved with math-magic has also been displayed and the ingredients for the spell has not been entered into the textboxes tagged x1 and x2.

In figure 3 below, the player has successfully plugged-in the right answers and his/her credit has increased by \$50 while a message was also displayed to that effect.

Finally, assuming a particular player could not solve the problem and desires to hire a sorcerer before his/her time runs out by clicking the "Hire a Sorcerer" button on the left bottom of the screen, a gallery of sorcerers will be displayed to the player as shown in figure 4 below:

Probability is computed within the game to determine if the sorcerer hired by the player will be able to solve that problem at that instance of time. If yes, the player earns the reward for solving the problem and also pays the sorcerer for the job done.

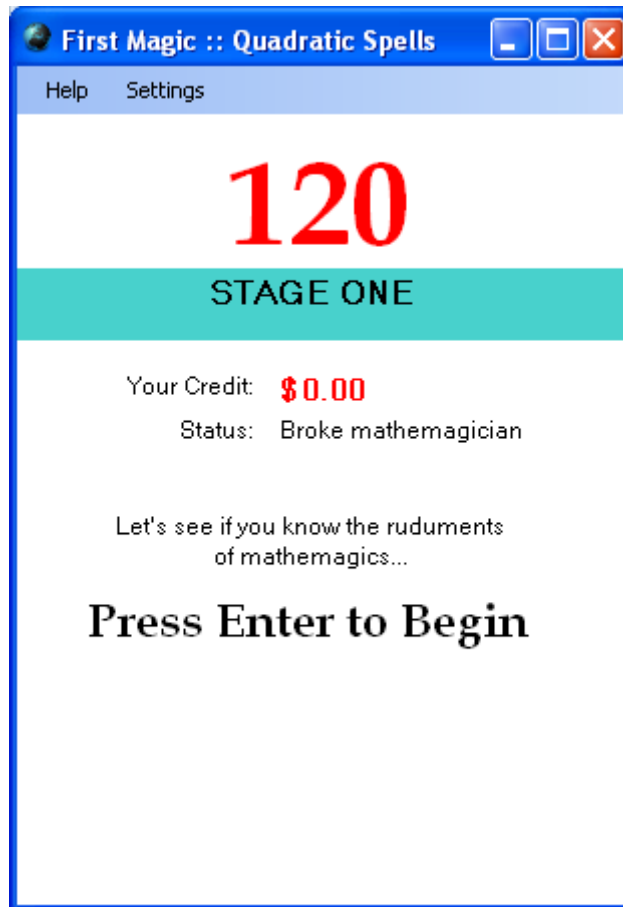


Figure 1: First Magic: Quadratic Spells

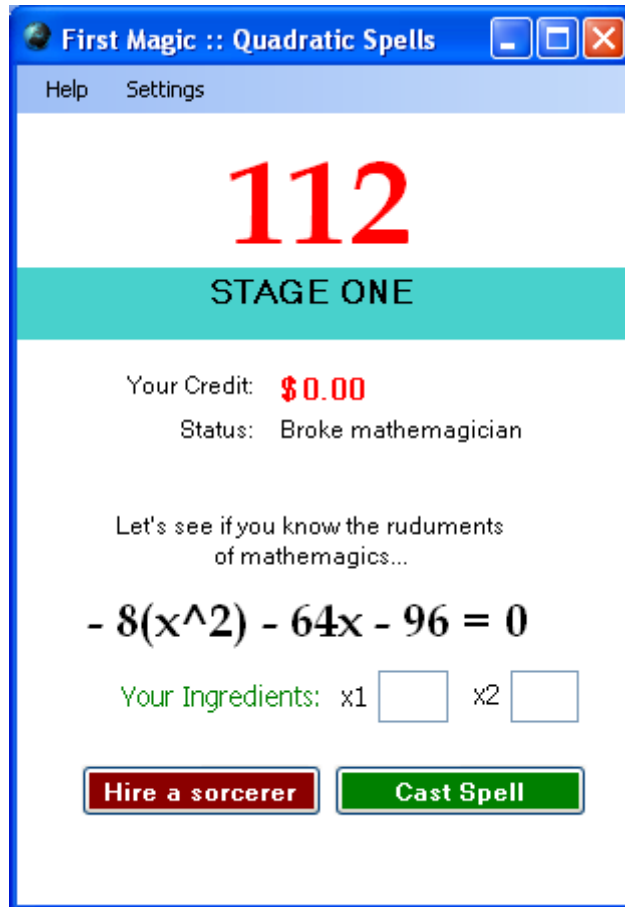


Figure 2: Casting the Spell to fix the first problem

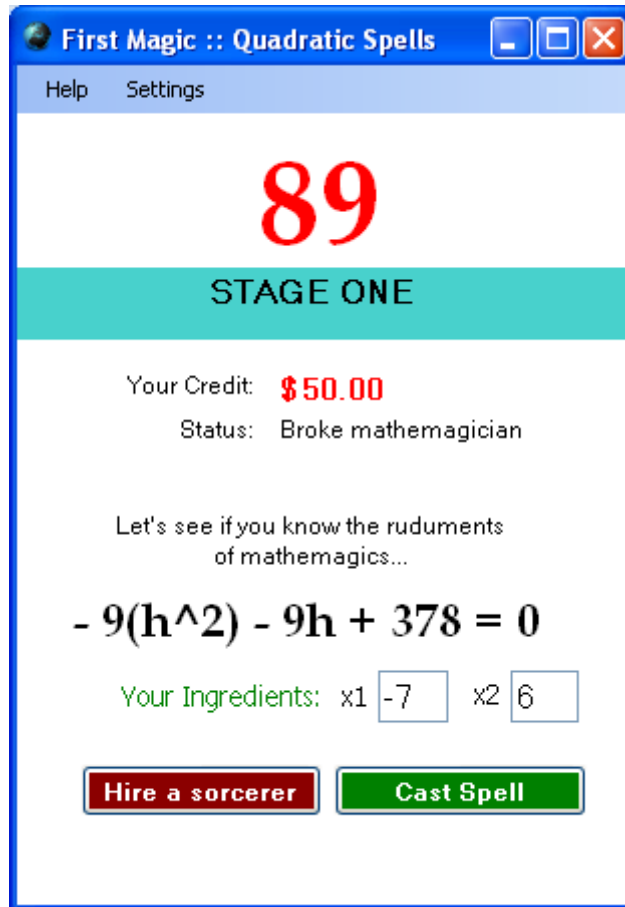


Figure 3: Player Scores a point



Figure 4

6. Conclusions

Our desire is to suggest here that during the play of this game, who gets to solve the problems is not of much interest to us but the struggle to earn more points is. This means that even if a player decides to hire the services of a professional mathematician to play the game, he/she would still be involved in one way or the other while our problems are simultaneously solved. With this, we declare that: getting the attention of the players and planting a strong addiction in their minds is our genuine goal.

However, in situations where anyone desires to compare students in mathematics using this game as a measurement scale, it will be necessary to have players under supervision.

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